QUANTUM GEOMETRODYNAMICAL DESCRIPTION OF THE UNIVERSE IN DIFFERENT REFERENCE FRAMES

T. P. Shestakova
Department of Theoretical and Computational Physics, Rostov State University,
Sorge Str. 5, Rostov-on-Don 344090, Russia
E-mail: stp@phys.rnd.runnet.ru

arXiv:gr-qc/0205068v1 16 May 2002

Abstract

Several years ago the so-called quantum geometrodynamics in extended phase space was proposed. The main role in this version of quantum geometrodynamics is given to a wave function that carries information about geometry of the Universe as well as about a reference frame in which this geometry is studied. We consider the evolution of a physical object (the Universe) in "physical" subspace of extended configurational space, the latter including gauge and ghost degrees of freedom. A measure of the "physical" subspace depends on a chosen reference frame, in particular, a small variation of a gauge-fixing function results in changing the measure. Thus, a transition to another gauge condition (another reference frame) leads to non-unitary transformation of a physical part of the wave function. From the viewpoint of the evolution of the Universe in the "physical" subspace a transition to another reference frame is an irreversible process that may be important when spacetime manifold has a nontrivial topology.

1 Introduction

Recently a new version of quantum geometrodynamics was proposed [1, 2]. While constructing this new version a special attention was paid to the fact that the Universe as a whole may not possess asymptotic states in which one can separate the so-called "non-physical" degrees of freedom from physical ones. In this report I shall refer to the case of a closed universe, but the same situation is expected to be in a general case if the Universe has some nontrivial topology.

In the path integral approach, which had been chosen to be a basic tool in our investigation, the lack of asymptotic states makes us refuse imposing asymptotic boundary conditions in a path integral. It leads to a gauge-dependent wave function of the Universe that satisfies a gauge-dependent Schrödinger equation. Indeed, in the modern quantum theory of gauge fields it is these very asymptotic boundary conditions that enable us to prove independence of a path integral on a chosen gauge (see, for example, [3]).

The goal of this report is to discuss some consequences of the proposed formulation of quantum geometrodynamics concerning the description of the Universe in different reference frames.

2 The new formulation of quantum geometrodynamics: basic equations

To make the things more clear, let us turn to a simple minisuperspace model with the gauged action

$$S=\int dt\left\{\frac{1}{2}v(\mu,Q^a)\gamma_{ab}\dot{Q}^a\dot{Q}^b-\frac{1}{v(\mu,Q^a)}U(Q^a)+\pi\left(\dot{\mu}-f_{,a}\dot{Q}^a\right)-iw(\mu,Q^a)\dot{\bar{\theta}}\dot{\theta}\right\}. \tag{1}$$

Here Q^a stands for physical variables such as a scale factor or gravitational-wave degrees of freedom and material fields, and we use an arbitrary parametrization of a gauge variable μ determined by the function $v(\mu,Q^a)$. For example, in the case of isotropic universe or the Bianchi IX model μ is bound to the scale factor r and the lapse function N by the relation

$$\frac{r^3}{N}=v(\mu,Q^a). \tag{2}$$

θ, $\bar{\theta}$ are the Faddeev – Popov ghosts after replacement $\bar{\theta}\to -i\bar{\theta}$. Further,

$$w(\mu,Q^a)=\frac{v(\mu,Q^a)}{v_{,\mu}};\quad v_{,\mu}\stackrel{def}{=}\frac{\partial v}{\partial\mu}. \tag{3}$$

We confine attention to the special class of gauges not depending on time

$$\mu=f(Q^a)+k;\quad k=\mathrm{const}, \tag{4}$$

which can be presented in a differential form,

$$\dot{\mu}=f_{,a}\dot{Q}^a,\quad f_{,a}\stackrel{def}{=}\frac{\partial f}{\partial Q^a}. \tag{5}$$

The Schrödinger equation for this model reads

$$i\,\frac{\partial\Psi(\mu,Q^a,\theta,\bar{\theta};\,t)}{\partial t}=H\Psi(\mu,\,Q^a,\,\theta,\,\bar{\theta};\,t), \tag{6}$$

where

$$H=-\frac{i}{w}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}}-\frac{1}{2M}\frac{\partial}{\partial Q^\alpha}MG^{\alpha\beta}\frac{\partial}{\partial Q^\beta}+\frac{1}{v}(U-V); \tag{7}$$

M is the measure in the path integral,

$$M(\mu,Q^a)=v^{\frac{K}{2}}(\mu,Q^a)w^{-1}(\mu,Q^a); \tag{8}$$

$$G^{\alpha\beta}=\frac{1}{v(\mu,Q^a)}\left(\begin{array}{cc} f_{,a}f^{,a} & f^{,a}\\ f^{,a} & \gamma^{ab}\end{array}\right);\quad \alpha,\beta=(0,a);\quad Q^0=\mu, \tag{9}$$

K is a number of physical degrees of freedom; the wave function is defined on extended configurational space with the coordinates μ, Q^a, θ, $\bar{\theta}$. V is a quantum correction to the potential U, that depends on the chosen parametrization (2) and gauge (4):

$$\begin{aligned}
V &= \frac{5}{12w^2}\left(w_{,\mu}^2 f_{,a}f^{,a}+2w_{,\mu}f_{,a}w^{,a}+w_{,a}w^{,a}\right)+\frac{1}{3w}\left(w_{,\mu,\mu}f_{,a}f^{,a}+2w_{,\mu,a}f^{,a}+w_{,\mu}f^{,a}_{,a}+w^{,a}_{,a}\right)+\\
&+ \frac{K-2}{6vw}\left(v_{,\mu}w_{,\mu}f_{,a}f^{,a}+v_{,\mu}f_{,a}w^{,a}+w_{,\mu}f_{,a}v^{,a}+v_{,a}w^{,a}\right)-\\
&- \frac{K^2-7K+6}{24v^2}\left(v_{,\mu}^2 f_{,a}f^{,a}+2v_{,\mu}f_{,a}v^{,a}+v_{,a}v^{,a}\right)+\\
&+ \frac{1-K}{6v}\left(v_{,\mu,\mu}f_{,a}f^{,a}+2v_{,\mu,a}f^{,a}+v_{,\mu}f^{,a}_{,a}+v^{,a}_{,a}\right).
\end{aligned} \tag{10}$$

Ones we agreed that imposing asymptotic boundary conditions is not correct in the case of a closed Universe, we *are doomed* to come to a gauge-dependent description of the Universe. The Schrödinger equation (6) – (10) is *a direct mathematical consequence* of a path integral with the effective action (1) without asymptotic boundary conditions, it is derived from the latter by the standard well-definite Feynman procedure. Any additional conditions like the requirement of BRST-invariance cannot help to reduce Eq. (6) to a gauge-invariant equation. So the role of such conditions is questionable when one deals with a system without asymptotic states.

3 The description of quantum Universe in different reference frames

The general solution to the Schrödinger equation (6) has the following structure:

$$\Psi(\mu,\,Q^a,\,\theta,\,\bar{\theta};\,t)=\int\Psi_k(Q^a,\,t)\,\delta(\mu-f(Q^a)-k)\,(\bar{\theta}+i\theta)\,dk. \tag{11}$$

The dependence of the wave function (11) on ghosts is determined by the demand of norm positivity.

Note that the general solution (11) is a superposition of eigenstates of a gauge operator,

$$\{\mu-f(Q^a)\}|k\rangle=k\,|k\rangle;\quad |k\rangle=\delta\left(\mu-f(Q^a)-k\right). \tag{12}$$

It can be interpreted in the spirit of Everett's "relative state" formulation. In fact, each element of the superposition (11) describe a state in which the only gauge degree of freedom μ is definite, so that time scale is determined by processes in the physical subsystem through functions $v(\mu,\,Q^a)$, $f(Q^a)$ (see (2), (4)), while k being determined initial clock setting. The

function $\Psi_k(Q^a,\,t)$ describes a state of the physical subsystem for a reference frame fixed by the condition (4). It is a solution to the equation

$$i\,\frac{\partial\Psi_k(Q^a;\,t)}{\partial t}=H_{(phys)}[f]\Psi_k(Q^a;\,t), \tag{13}$$

$$H_{(phys)}[f]=\left[-\frac{1}{2M}\frac{\partial}{\partial Q^a}\frac{1}{v}M\gamma^{ab}\frac{\partial}{\partial Q^b}+\frac{1}{v}(U-V)\right]\Bigg|_{\mu=f(Q^a)+k}. \tag{14}$$

The peculiarity of this consideration is that a measure in the subspace of physical degrees of freedom depends on a chosen gauge condition. Indeed, the measure (8) in the path integral is proportional to a square root of the determinant of metric of “physical” configurational subspace, the latter depending on the gauge variable μ: $G^{phys}_{ab}=v(\mu,Q^a)$. So we get

$$\int\Psi^*_{k'}(Q^a,\,t)\,\Psi_k(Q^a,\,t)\,\delta(\mu-f(Q^a)-k')\,\delta(\mu-f(Q^a)-k)\,dk'\,dk\,M(\mu,\,Q^a)\,d\mu\prod_a dQ^a=$$

$$=\int\Psi^*_k(Q^a,\,t)\,\Psi_k(Q^a,\,t)\,M(f(Q^a)+k,\,Q^a)\prod_a dQ^a\,dk=1. \tag{15}$$

It is easy to see that a transition to another gauge condition (another reference frame) cannot be described by an unitary transformation of the physical part of the wave function $\Psi_k(Q^a,\,t)$. As a consequence of this structure of physical subspace, we will obtain different physical results in different reference frames.

One can seek the solution to Eq.(13) in the form of superposition of stationary state eigenfunctions:

$$\Psi_k(Q^a,\,t)=\sum_n c_{kn}\psi_n(Q^a)\exp(-iE_nt);\quad H_{(phys)}[f]\psi_n(Q^a)=E_n\psi_n(Q^a). \tag{16}$$

The parameter E should not be associated with energy of any material field. It is a new integral of motion that emerges in the proposed formulation as a result of fixing a gauge condition and characterizes a subsystem which corresponds to observation means – a reference frame (see [1, 2] for details).

The proposed formulation of quantum geometrodynamics suffers from the fact that, according to it, the Universe could be created in any state with a nonzero value of the parameter E. On the other hand, we can surely say that at the present stage of its evolution the Universe is found in the state with $E=0$. For this state only we can obtain a gauge-invariant classical limit. So we need some mechanism of the “reduction” of the wave function to the state with $E=0$.

We also do not know a criterion for a choice of a reference frame. While we do not know any deeper reason, our choice may be dictated by convenience and simplicity. However, it is

rather rare situation when a spacetime manifold can be covered by only one coordinate system. In a case of nontrivial topology spacetime may consist of several regions covered by different coordinate charts. It is a serious problem for the Dirac canonical quantization which requires to introduce a foliation by spacelike hypersurfaces that would cover all available spacetime. The approach presented here which is based on path integration may turn to be more adequate for description of this situation, since the path integral admits (at least formally) introducing different gauge conditions in different spacetime regions.

Then, in this approach, the wave function may satisfy different Schrödinger equations in different spacetime regions, the form of the equation in each region is fixed by a chosen reference frame. Like a transition to another reference frame in the same spacetime region, a transition from one spacetime region to another must be described by a non-unitary transformation of the wave function.

The problem arises if it is possible to give a mathematical description to the transition to a different reference frame. We can try to do it for our minisuperspace model where the only role of a gauge condition is in fixing time scale.

We now consider the equation for the physical part of the wave function when varying the gauge-fixing function $f(Q^a)$. Let a reference frame be fixed by the condition

$$\mu = f(Q^a) + \delta f(Q^a) + k. \tag{17}$$

We can choose a basis corresponding to this reference frame, so that

$$\Psi(\mu,\, Q^a,\, \theta,\, \bar{\theta};\, t) = \int \tilde{\Psi}_k(Q^a,\, t)\, \delta(\mu - f(Q^a) - \delta f(Q^a) - k)\, (\bar{\theta} + i\theta)\, dk. \tag{18}$$

The function $\tilde{\Psi}_k(Q^a,\, t)$ satisfies Eq.(13) with a Hamiltonian

$$H_{(phys)}[f + \delta f] = \left[-\frac{1}{2M}\frac{\partial}{\partial Q^a}\left(\frac{1}{v}M\gamma^{ab}\frac{\partial}{\partial Q^b}\right) + \frac{1}{v}(U - V)\right]\Bigg|_{\mu = f(Q^a) + \delta f(Q^a) + k}. \tag{19}$$

If the variation of the gauge-fixing function $\delta f(Q^a)$ is small, one can write

$$H_{(phys)}[f + \delta f] = H_{(phys)}[f] + W[\delta f] + V_1[\delta f] \tag{20}$$

For our minisuperspace model the operator $W[\delta f]$ reads

$$\begin{aligned} W[\delta f] \;&=\; \left[\frac{1}{2M^2}\frac{\partial M}{\partial \mu}\delta f \frac{\partial}{\partial Q^a}\left(\frac{1}{v}M\gamma^{ab}\frac{\partial}{\partial Q^b}\right) - \right. \\ &- \left.\frac{1}{2M}\frac{\partial}{\partial Q^a}\left(\left(\frac{1}{v}\frac{\partial M}{\partial \mu} - \frac{M}{v^2}\frac{\partial v}{\partial \mu}\right)\delta f \gamma^{ab}\frac{\partial}{\partial Q^b}\right)\right]\Bigg|_{\mu = f(Q^a) + k}, \end{aligned} \tag{21}$$

and $V_1[\delta f]$ is the change of quantum potential V in first order of δf.

We can inquire how the probabilities of states (16) change under the perturbation $W[\delta f] + V_1[\delta f]$, which is due to a small variation of the gauged-fixing function $f(Q^a)$. The Hamiltonian (19) is Hermitian by construction in a space with the measure $M(f(Q^a) + \delta f(Q^a) + k,\, Q^a)$, however it is not Hermitian in a space with the measure $M(f(Q^a)+k,\, Q^a)$ in which the functions (16) are normalized. In this space the operator (21) will have, in general, anti-Hermitian part. So it follows already from Eqs. (19) – (21) that a transition to another reference frame is *an irreversible process*.

This conclusion is in accordance with our interpretation of the reference frame as the only measuring instrument representing the observer in quantum theory of gravity. The variation of a gauge-fixing function means changing interaction with the measuring instrument that implies an unremoval influence on properties of the physical object.

References

[1] Savchenko V. A., Shestakova T. P. and Vereshkov G. M. 2001 *Gravitation & Cosmology* **7**, 18.
[2] Savchenko V. A., Shestakova T. P. and Vereshkov G. M. 2001 *Gravitation & Cosmology* **7**, 102.
[3] Hennaux M. 1985, *Phys. Rep.* **126**, 1.